Topic 2

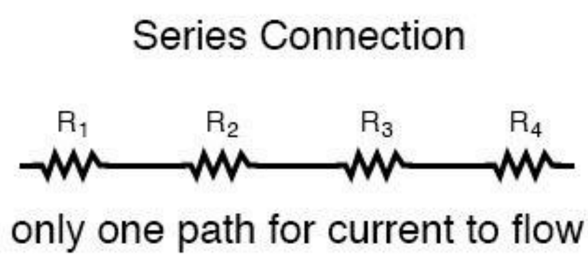
Equivalent resistance, voltage-current divider rule, delta-star transformations

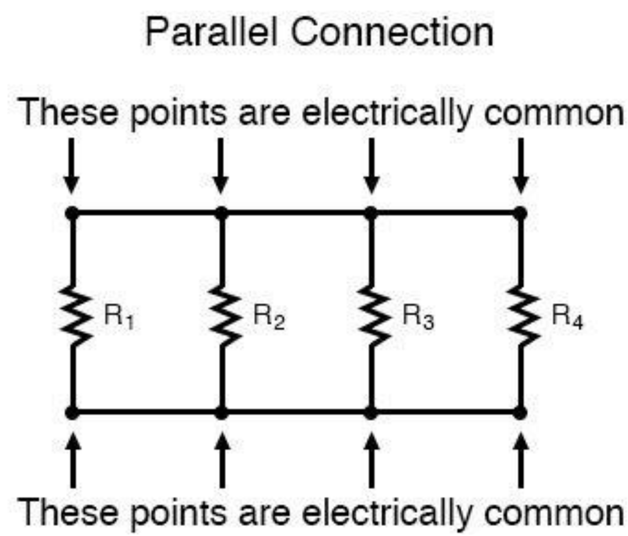
*Prepared by, **Dr. A.S.M. Iftekhar Uddin***

Connections

- Series
- Parallel
- Ladder or combined

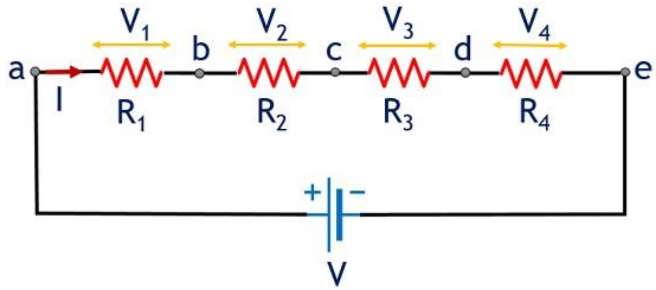
Individual resistors can be connected together in either a series connection, a parallel connection or combinations of both series and parallel, to produce more complex resistor networks whose equivalent resistance is the mathematical combination of the individual resistors connected together.





- In a series circuit, all components are connected end-to-end, forming a single path for current flow.
- In a parallel circuit, all components are connected across each other, forming exactly two sets of electrically common points.

Series Connection



Series Circuit

Equivalent resistance, $R_T = R_1 + R_2 + R_3 \ldots\ldots\ldots + R_n$

Example:

Equivalent resistance for the series arrangement (fig. a),

$R_T = R_1 + R_2 + R_3$

$R_T = 15 + 20 + 15 = 50\ \Omega$

$I = E/R_T = 14/50 = 0.28\ A$ (applying Ohm's Law)

$V_1 = IR_1 = 0.28 \times 15 = 4.2\ V$ (applying Ohm's Law)

$V_2 = IR_2 = 0.28 \times 20 = 5.6\ V$ (applying Ohm's Law)

$V_3 = IR_3 = 0.28 \times 15 = 4.2\ V$ (applying Ohm's Law)

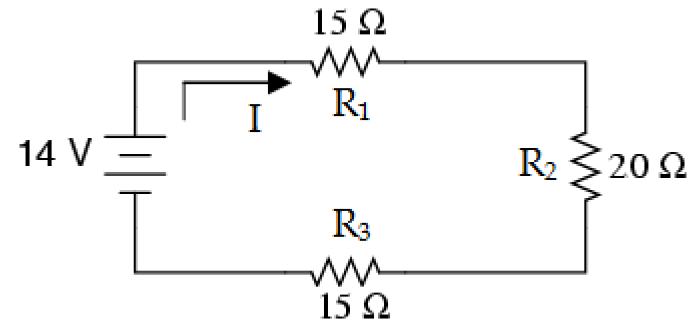
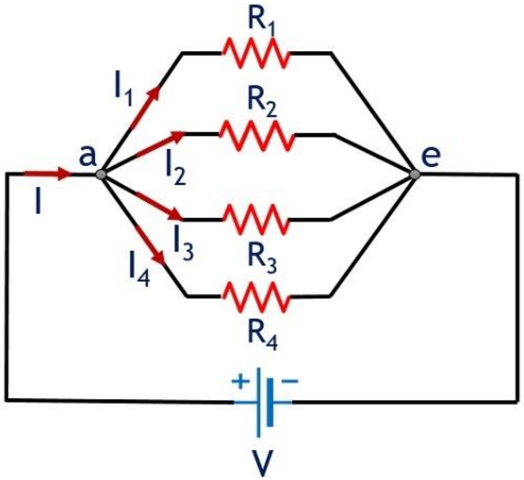


Figure: (a)

Parallel Connection

Equivalent resistance, $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} .... + \frac{1}{R_n}$



Parallel Circuit

Example:

Equivalent resistance for the parallel arrangement (fig. b),

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{1}{5} + \frac{1}{3} + \frac{1}{6} = \frac{6+10+5}{30} = \frac{21}{30}$$

$$R_T = \frac{30}{21} = 1.43\Omega$$

$I = E/R_T = 20/1.43 = 14$ A

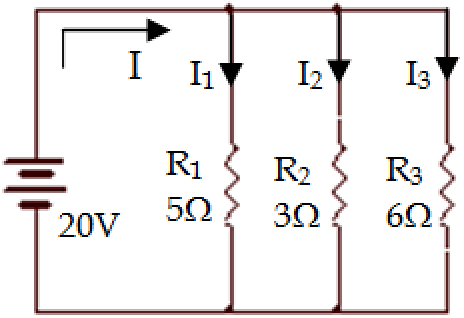


Figure: (b)

Series-Parallel Connection

Example:

Equivalent resistance for the series – parallel arrangement (fig. c),

$$R_T = R_1 + R_2 \parallel R_3 = 4\Omega + 2\Omega \parallel 1\Omega = 4 + 0.67 = 4.67\Omega$$

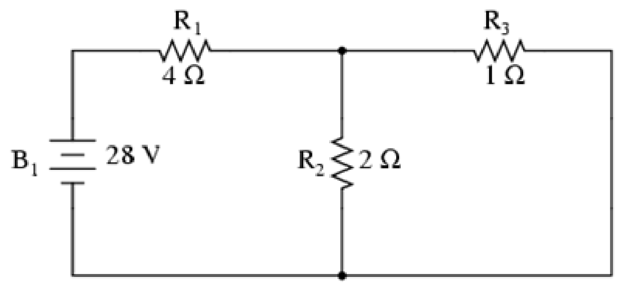
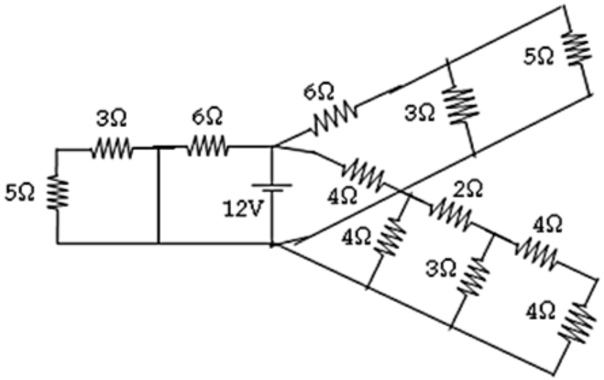


Figure: (c)

Problem 1: Calculate the equivalent resistance of the following circuit shown below,



Solution 1:

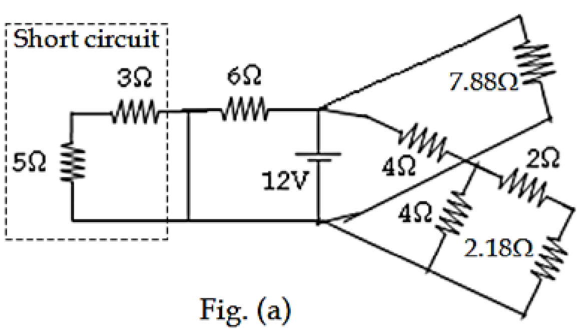


Fig. (a)

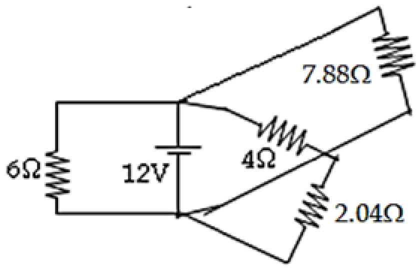


Fig. (b)

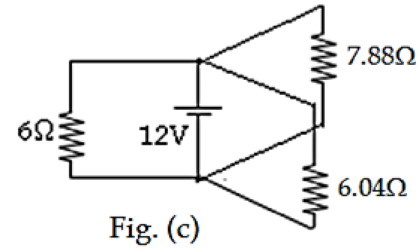


Fig. (c)

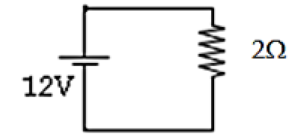
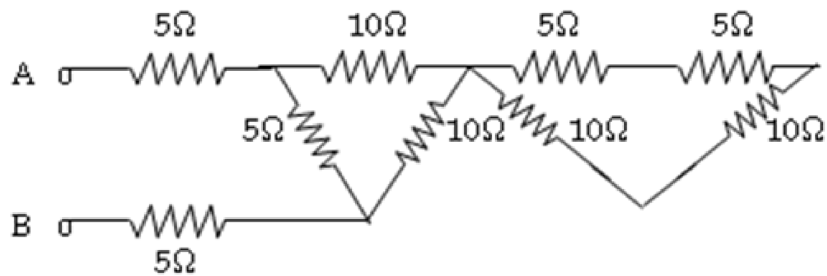
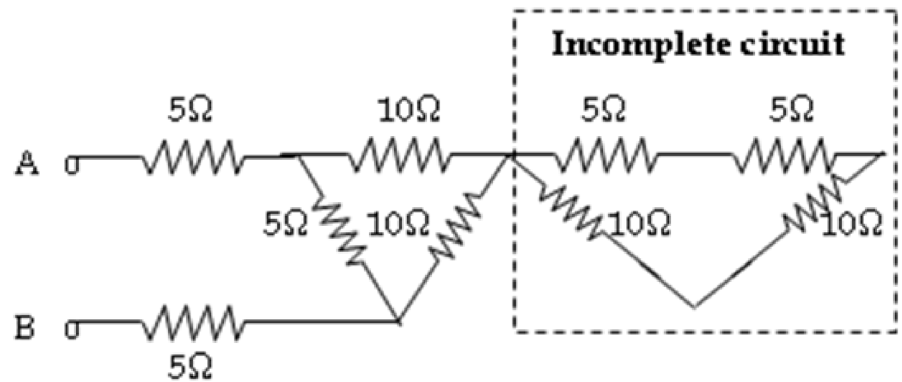


Fig. (d)

Problem 2: Calculate the equivalent resistance of the following circuit shown below,



Solution 2:



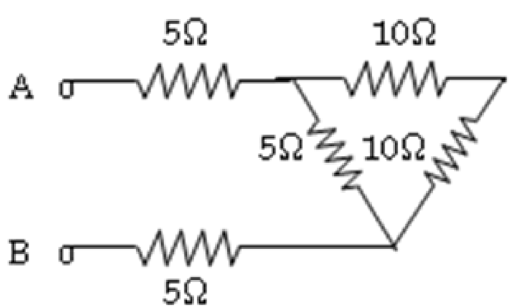


Fig. (a)

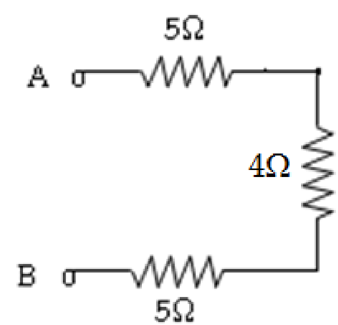


Fig. (b)

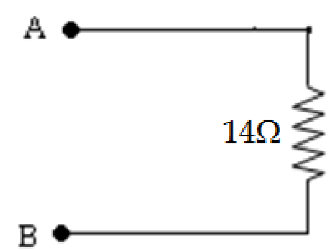


Fig. (c)

Voltage-Current divider rule

VOLTAGE DIVIDER RULE:

In fig. (a),

$$V_1 = \frac{E \times R_1}{R_1 + R_2 + R_3} = \frac{14 \times 15}{15 + 20 + 15} = \frac{210}{50} = 4.2V$$

$$V_2 = \frac{E \times R_2}{R_1 + R_2 + R_3} = \frac{14 \times 20}{15 + 20 + 15} = \frac{280}{50} = 5.6V$$

$$V_3 = \frac{E \times R_3}{R_1 + R_2 + R_3} = \frac{14 \times 15}{15 + 20 + 15} = \frac{210}{50} = 4.2V$$

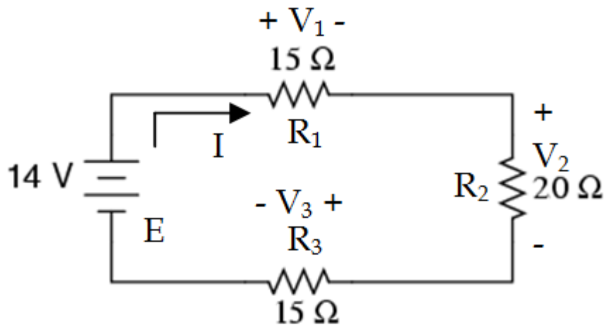


Figure: (a)

CURRENT DIVIDER RULE:

In fig. (b),

$$R_T = R_1 \parallel R_2 = 5 \parallel 7 = 2.92\Omega$$

$$\therefore I = \frac{E}{R_T} = \frac{20}{2.92} = 6.85A$$

So,

$$I_1 = \frac{I \times R_2}{R_1 + R_2} = \frac{6.85 \times 7}{5 + 7} = \frac{47.95}{12} = 4A$$

$$I_2 = \frac{I \times R_1}{R_1 + R_2} = \frac{6.85 \times 5}{5 + 7} = \frac{34.25}{12} = 2.85A$$

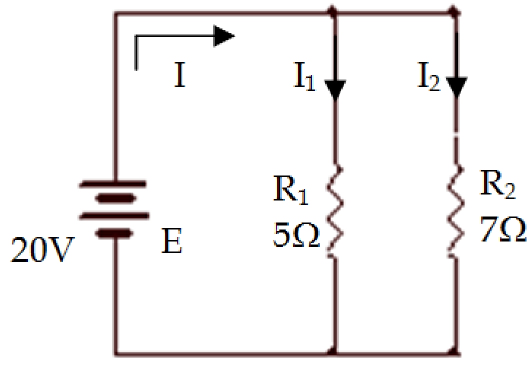
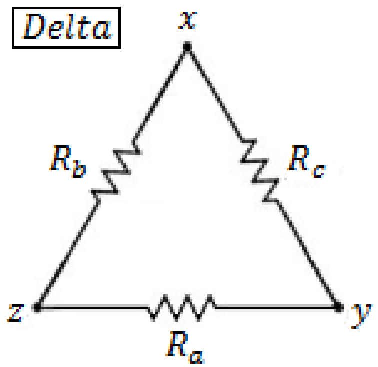
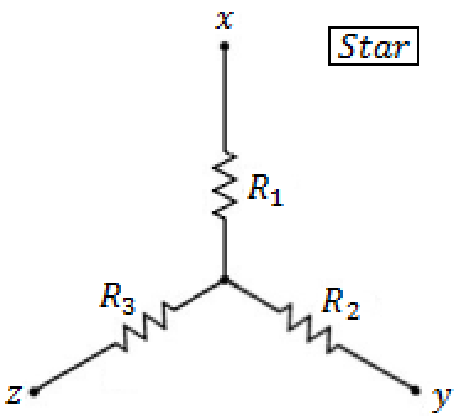


Figure: (b)

Delta-Star transformations





$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

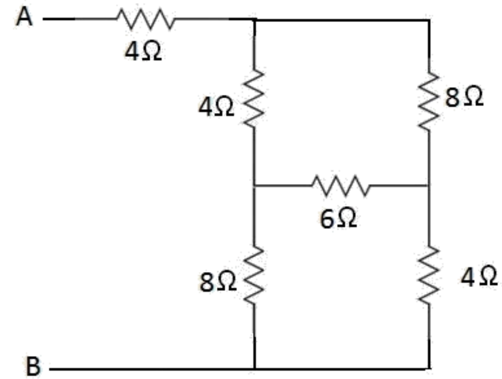
$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_1 = \frac{R_bR_c}{R_a + R_b + R_c}$$

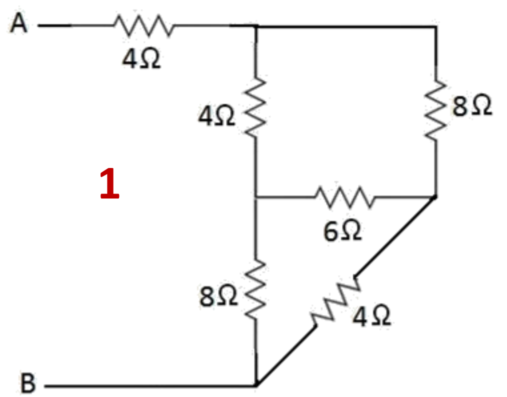
$$R_2 = \frac{R_aR_c}{R_a + R_b + R_c}$$

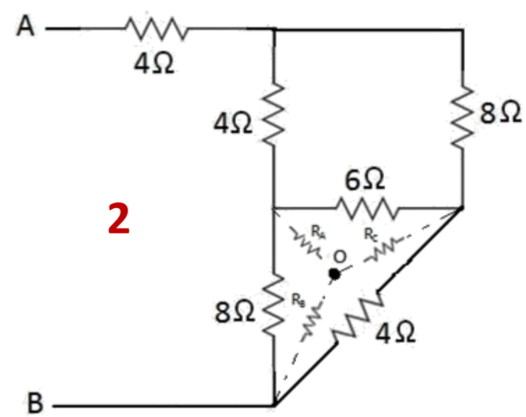
$$R_3 = \frac{R_aR_b}{R_a + R_b + R_c}$$

Problem 3: Calculate the equivalent resistance of the following circuit shown below,



Solution 2:





$$R_A = \frac{8 \cdot 6}{8 + 6 + 4}$$

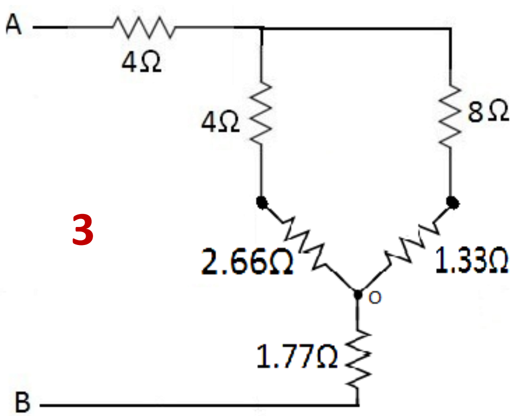
$$R_A = 2.66\Omega$$

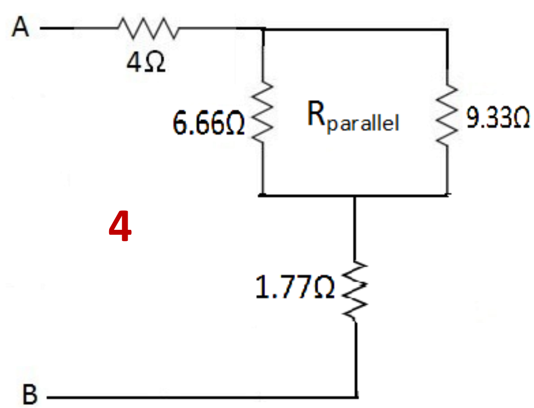
$$R_B = \frac{8 \cdot 4}{8 + 6 + 4}$$

$$R_B = 1.77\Omega$$

$$R_C = \frac{6 \cdot 4}{8 + 6 + 4}$$

$$R_C = 1.33\Omega$$





$$R_{parallel} = \frac{6.66 \times 9.33}{6.66 + 9.33} = 3.88\Omega$$